Project #10

Properties of Continuous Time Fourier Series, Linearity

"Linearity property" of Fourier series (FS) representation can be expressed as follows

$$x(t) \xleftrightarrow{FS} a_k,\ y(t) \xleftrightarrow{FS} b_k \Rightarrow Ax(t) + By(t) \xleftrightarrow{FS} Aa_k + Bb_k, \quad (10.1)$$

where $x(t)$ and $y(t)$ are two periodic signals with the same period and have a_k and b_k as their FS coefficients respectively. As the statement (10.1) indicates, we can use this property in evaluating FS coefficients of a periodic signal that can be expressed as a linear combination of other periodic signals whose FS coefficients are known.

We will demonstrate the validity of this property using two periodic square wave signals, $x(t)$ and $y(t)$, with $T = 1$ and different T_1 parameters.

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < T/2 \end{cases},\ x(t) = x(t+T),\ T = 1 \text{ and } T_1 = 1/4 \quad (10.2)$$

$$y(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < T/2 \end{cases},\ y(t) = y(t+T),\ T = 1 \text{ and } T_1 = 1/8 \quad (10.3)$$

These periodic signals are shown in Figure 10.1. Matlab code segments of this project are presented at the end, in an mfile titled "project10.m". In this code, we first generated the two periodic square wave signals. We generated the first one by using "thresholding cosine technique" that we introduced in Project #9. For the second one, we generated a period of it and repeated that period or segment 3 times. (Can you tell why we cannot generate this second signal with $T_1 = 1/8$ using thresholding cosine technique?) Then, we found the FS representation or FS coefficients these signals using formula (9.2). Finally, we added those coefficients a_k's and b_k's to obtain c_k's from which we made a reconstruction using 101 terms. The resulting signal is shown in Figure 10.2. Does it look like $x(t) + y(t)$? Instead of second square wave $y(t)$, use a 1Hz cosine, which has a simple FS representation, and redo the project.

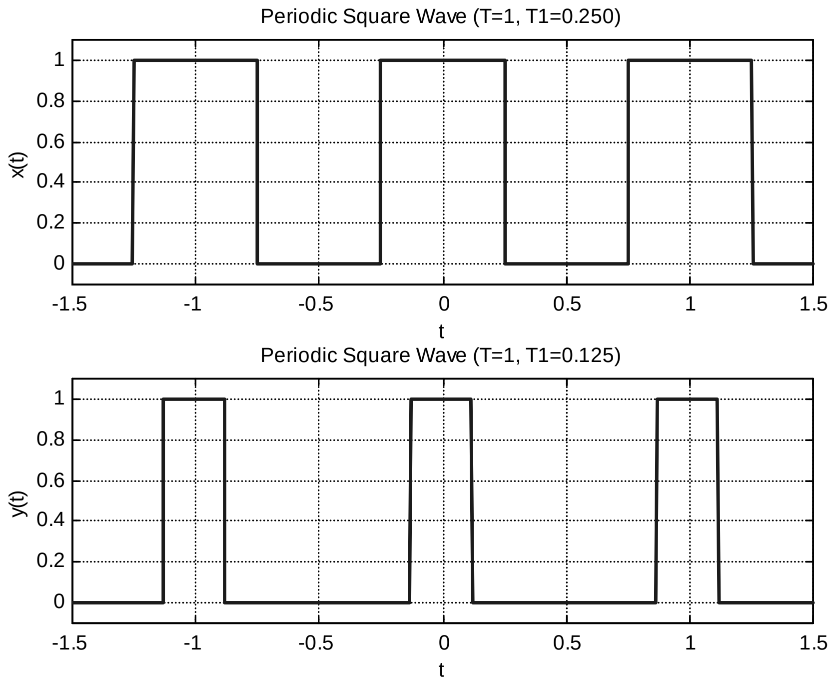

Figure 10.1 Two periodic square wave signals with $T = 1$ and $T_1 = 0.25$ (upper panel) and $T = 1$ and $T_1 = 0.125$ (lower panel).

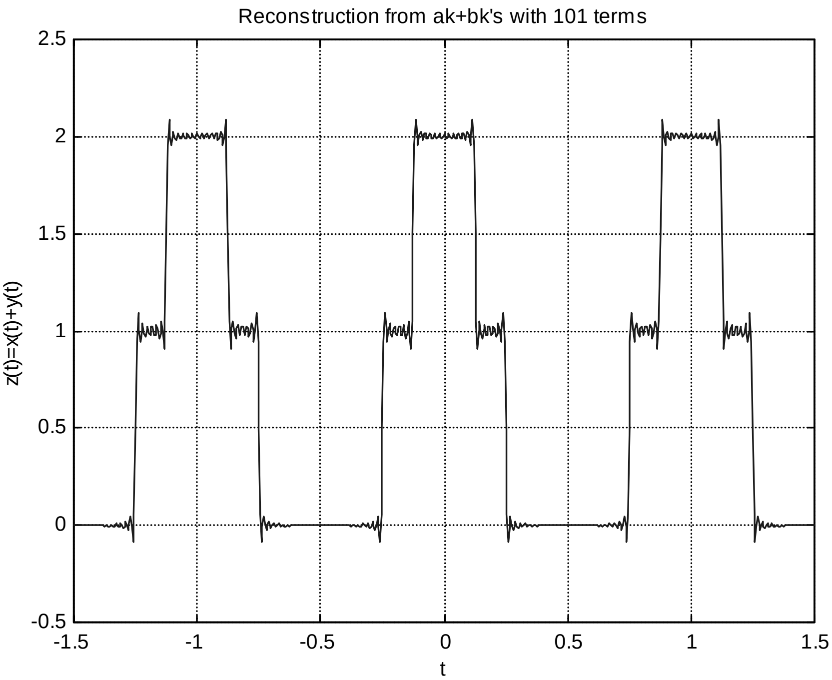

Figure 10.2 Reconstruction using coefficients $c_k = a_k + b_k$'s with 101 terms ($k = -50 .. 50$).

```
% Project #10
% Title: Properties of Continuous Time Fourier Series, Linearity

% Generation of 1st signal xt
t=-1.5:0.005:1.5;
xcos=cos(2*pi*t);
xt=xcos>0;

figure(1);set(gcf,'defaultaxesfontsize',9)
subplot(2,1,1);plot(t,xt);xlabel('t');ylabel('x(t)')
title('Peridoic Square Wave (T=1, T1=0.250)')
set(gca,'ylim',[-0.1 1.1]);grid

% Generation of 2nd signal yt
T=1;T1=0.125;
lenT=T/0.005;
ytemp=zeros(1,lenT);
lenT1=T1/0.005;
ytemp(round(lenT/2)-lenT1:round(lenT/2)+lenT1-1)=ones(1,2*lenT1);

yt=[ytemp ytemp ytemp 0];
% The last 0 added to make the size of yt equal to length(t)

subplot(2,1,2);plot(t,yt);xlabel('t');ylabel('y(t)')
title('Periodic Square Wave (T=1, T1=0.125)')
set(gca,'ylim',[-0.1 1.1]);grid

% FS coefficients of periodic square waves
k=-50:50;

T1=0.25;
ak=sin(k*2*pi*(T1/T))./(k*pi);
% Manual correction for a0 -> ak(51)
ak(51)=2*T1/T;

T1=0.125;
bk=sin(k*2*pi*(T1/T))./(k*pi);
```

```
% Manual correction for b0 -> bk(51)
bk(51)=2*T1/T;

% Application of linearity property of FS
ck=ak+bk;

% Reconstruction with M=50
w0=2*pi/T;
zt=zeros(1,length(t));
for k=-50:50
   zt=zt + ck(k+51)*exp(j*k*w0*t);
end

figure(2);set(gcf,'defaultaxesfontsize',9)
plot(t,real(zt));grid;xlabel('t');ylabel('z(t)=x(t)+y(t)')
title('Reconstruction from ak+bk''s with 101 terms')
```