Project #11

Properties of Continuous Time Fourier Series, Time Shifting

In this project we will demonstrate the validity of "time shifting property" of Fourier series (FS). Again we will work with a periodic square wave signal to construct our example, as we already know the FS representation, or a_k's, for square waves.

Time shifting property of FS states that

$$x(t) \xleftrightarrow{FS} a_k \Leftrightarrow x(t-t_0) \xleftrightarrow{FS} e^{-jkw_0t_0}a_k,\ w_0 = 2\pi/T\ , \qquad (11.1)$$

where $x(t)$ is a periodic signal with period T that has a_k's as its FS coefficients and $x(t-t_0)$ is the time shifted version of it. We can use this property in evaluating FS coefficients of a periodic signal that can be expressed as time shifted version another periodic signal whose FS coefficients are known.

For a periodic square wave with $T = 1$ and $T_1 = 0.25$, we already know that FS coefficients a_k's are

$$a_0 = \frac{2T_1}{T} = 0.5, \quad a_k = \frac{\sin(k2\pi(T_1/T))}{k\pi} = \frac{\sin(k\pi/2)}{k\pi} \text{ for } k \neq 0. \qquad (11.2)$$

Let time shift $t_0 = 0.25$, FS coefficients b_k's for $x(t-t_0) = x(t-0.25)$ can be found by using (11.1) and (11.2) as

$$b_k = e^{-jkw_0t_0}a_k, \quad t_0 = 0.25, \quad w_0 = 2\pi/T = 2\pi\ . \qquad (11.3)$$

Finally, we made a reconstruction using 101 of these coefficients, i.e. b_k's for $k = -50$ .. 50. The resulting signal is shown in Figure 11.1. Does it look like $x(t-0.25)$, the periodic square wave shifted in time by 0.25?

You can find Matlab code segments of this project in mfile "project11.m" given below.

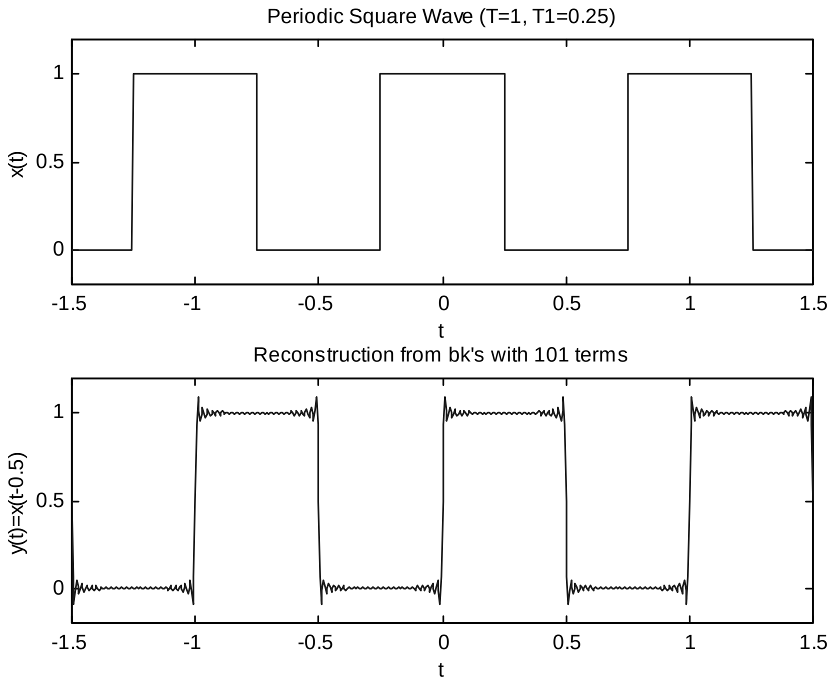

Figure 11.1 Periodic square wave signal with $T = 1$ and $T_1 = 0.25$ (upper panel) and $T = 1$ and reconstructed signal using coefficients b_k's for $k = -50$ .. 50 (lower panel).

```
% Project #11
% Title: Properties of Continuous Time Fourier Series
%        Time Shifting

% Generation of periodic square wave
t=-1.5:0.005:1.5;
xcos=cos(2*pi*t);
xt=xcos>0;

% FS coefficients of periodic square wave
k=-50:50;
T=1;T1=0.25;
ak=sin(k*2*pi*(T1/T))./(k*pi);
% Manual correction for a0 -> ak(51)
```

```
ak(51)=2*T1/T;

% Amount of time shift
t0=0.25;

% FS coefficients of the time shifted signal
w0=2*pi/T;
bk=ak.*exp(-j*k*w0*t0);

% Reconstruction from bk's with 101 terms (M=50)
yt=zeros(1,length(t));
for k=-50:50
   yt=yt + bk(k+51)*exp(j*k*w0*t);
end

figure(1);set(gcf,'defaultaxesfontsize',9)
subplot(2,1,1);plot(t,xt);xlabel('t');ylabel('x(t)')
title('Periodic Square Wave (T=1, T1=0.25)')
set(gca,'ylim',[-0.2 1.2]);grid

subplot(2,1,2);plot(t,real(yt))
xlabel('t');ylabel('y(t)=x(t-0.5)')
title('Reconstruction from bk''s with 101 terms')
set(gca,'ylim',[-0.2 1.2]);grid
```