Project #12

Properties of Continuous Time Fourier Series, Time Reversal

In this project we will demonstrate the validity of "time reversal property" of Fourier series (FS), which can be expressed as follows.

$$x(t) \overset{FS}{\longleftrightarrow} a_k \Leftrightarrow x(-t) = \overset{FS}{\longleftrightarrow} a_{-k} \tag{12.1}$$

If FS representation for a periodic signal $x(t)$ is known, then FS representation for the time reversed version of the signal $x(-t)$ is readily available through this property.

Again, we will work with a periodic square wave signal to construct our example but this time the signal will be an odd square wave as shown in Figure 12.1 (upper panel). We obtained this same signal in Project #11 by time shifting a periodic square wave with $T = 1$ and $T_1 = 0.25$, therefore we already know its FS representation. From equations (11.2) and (11.3), coefficients of this signal are

$$\begin{aligned} a_0 &= \frac{2T_1}{T} = 0.5, \\ a_k &= e^{-jkw_0t_0}\frac{\sin(k\pi/2)}{k\pi} \text{ for } k \neq 0,\ \ t_0 = 0.25,\ \ w_0 = 2\pi/T = 2\pi. \end{aligned} \tag{12.1}$$

To see whether these coefficients will really give rise to our odd periodic square wave signal, we made a reconstruction using 101 of these coefficients, i.e. a_k's for $k = -50$ .. 50, the resulting signal is shown in Figure 12.1 (middle panel).

Finally, we reversed or flipped these coefficients to obtain b_k's as

$$b_k = a_{-k}, \tag{12.2}$$

and made a reconstruction using 101 of these coefficients, i.e. b_k's for $k = -50$ .. 50. The resulting signal is shown in Figure 12.1 (lower panel).

As we see, this last signal $y(t)$ is the time reversed version of our original signal $x(t)$, i.e. $y(t) = x(-t)$.

Matlab code of this project is given below, in mfile "project12.m".

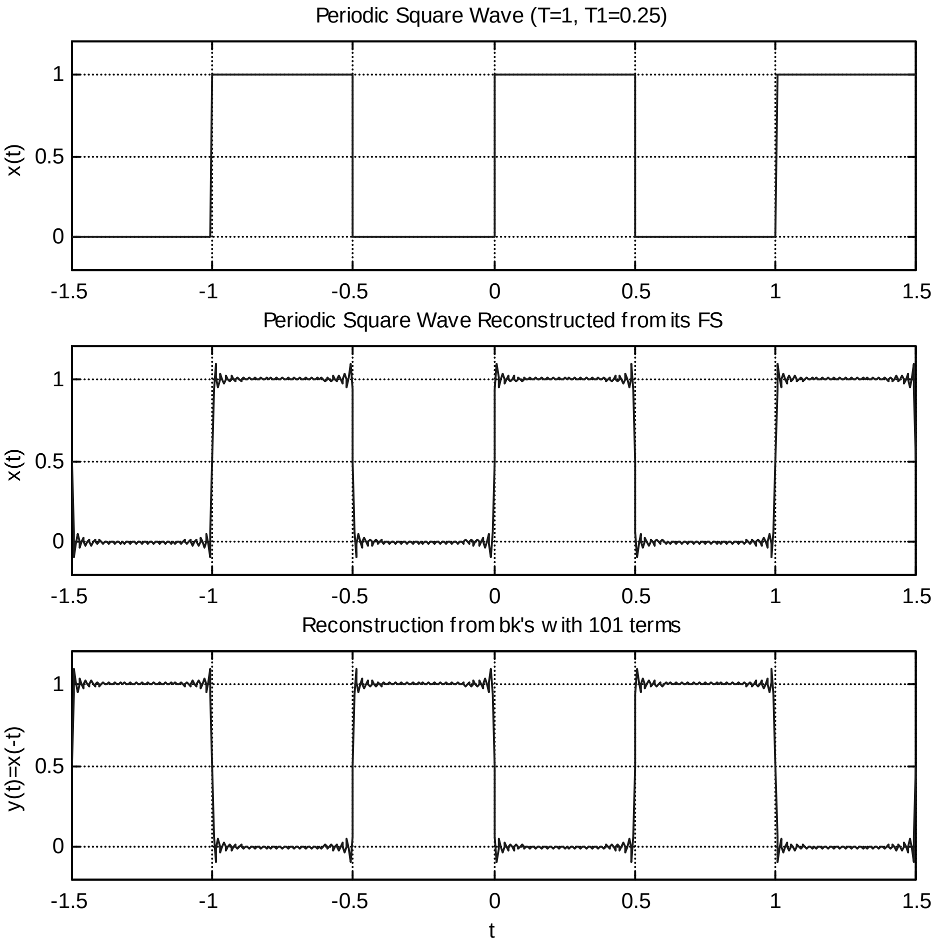

Figure 12.1 An odd, periodic square wave signal ($T = 1$, $T_1 = 0.25$, time shift $t_0 = 0.25$) (upper panel), reconstruction of this signal from its FS using 101 coefficients (middle panel), and reconstruction from flipped FS coefficients (lower panel).

```
% Project #12
% Title: Properties of Continuous Time Fourier Series
%        Time Reversal

% Generation of periodic square wave
t=-1.5:0.005:1.5;
xsin=sin(2*pi*t);
xt=xsin>0;
```

```
% FS coefficients of periodic square wave
k=-50:50;
T=1;T1=0.25;w0=2*pi/T;t0=0.25;
ak=exp(-j*k*w0*t0).*sin(k*2*pi*(T1/T))./(k*pi);

% Manual correction for a0 -> ak(51)
ak(51)=2*T1/T;

% FS coefficients of the time reversed signal
bk=fliplr(ak);

% Reconstruction from ak's with 101 terms (M=50)
xtr=zeros(1,length(t));
for k=-50:50
   xtr=xtr + ak(k+51)*exp(j*k*w0*t);
end

% Reconstruction from bk's with 101 terms (M=50)
yt=zeros(1,length(t));
for k=-50:50
   yt=yt + bk(k+51)*exp(j*k*w0*t);
end

figure(1);set(gcf,'defaultaxesfontsize',8)
subplot(3,1,1);plot(t,xt);ylabel('x(t)')
title('Periodic Square Wave (T=1, T1=0.25)')
set(gca,'ylim',[-0.2 1.2]);grid

subplot(3,1,2);plot(t,xtr);ylabel('x(t)')
title('Periodic Square Wave Reconstructed from its FS')
set(gca,'ylim',[-0.2 1.2]);grid

subplot(3,1,3);plot(t,real(yt))
xlabel('t');ylabel('y(t)=x(-t)')
title('Reconstruction from bk''s with 101 terms')
set(gca,'ylim',[-0.2 1.2]);grid
```