Project #13

Properties of Continuous Time Fourier Series, Time Scaling

We intuitively feel that "time scaling" a periodic signal should also produce a periodic signal. We can see the effect of time scaling on a periodic signal by using the reconstruction equation.

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jkwt} \Rightarrow x(\alpha t) = \sum_{k=-\infty}^{+\infty} a_k e^{jkw\alpha t} = \sum_{k=-\infty}^{+\infty} a_k e^{jk(\alpha w)t} \qquad (13.1)$$

From equation (13.1) we deduce that, if the signal $x(t)$ is periodic with period T and fundamental frequency $w = 2\pi / T$, then time scaled version of $x(t)$, $x(\alpha t)$, α being positive real number, will have αw as its fundamental frequency and the Fourier series (FS) coefficients for $x(\alpha t)$ will be same as those of $x(t)$. The only point to be careful about is to use the right, i.e. scaled, frequency (or period) in the reconstruction.

We can express this observation mathematically as follows.

$$\begin{array}{ccc} x(t) \overset{FS}{\longleftrightarrow} a_k & \Leftrightarrow & x(\alpha t) \overset{FS}{\longleftrightarrow} a_k \\ x(t) = x(t+T), w = \dfrac{2\pi}{T} & & x(\alpha t) = x(\alpha(t + \dfrac{T}{\alpha})), w = \alpha \dfrac{2\pi}{T} \end{array} \qquad (13.2)$$

We will demonstrate validity of this property using the familiar periodic square wave signal with a well known FS representation. Figure 13.1 shows this signal along with two reconstructed signals from its FS coefficients. As we see, in the first reconstruction, Figure 13.1 (middle panel), choosing α as 0.5 has doubled the period and in the second reconstruction, Figure 13.1 (lower panel), choosing α as 2 has halved the period.

You can find Matlab code of this project below, in mfile "project13.m".

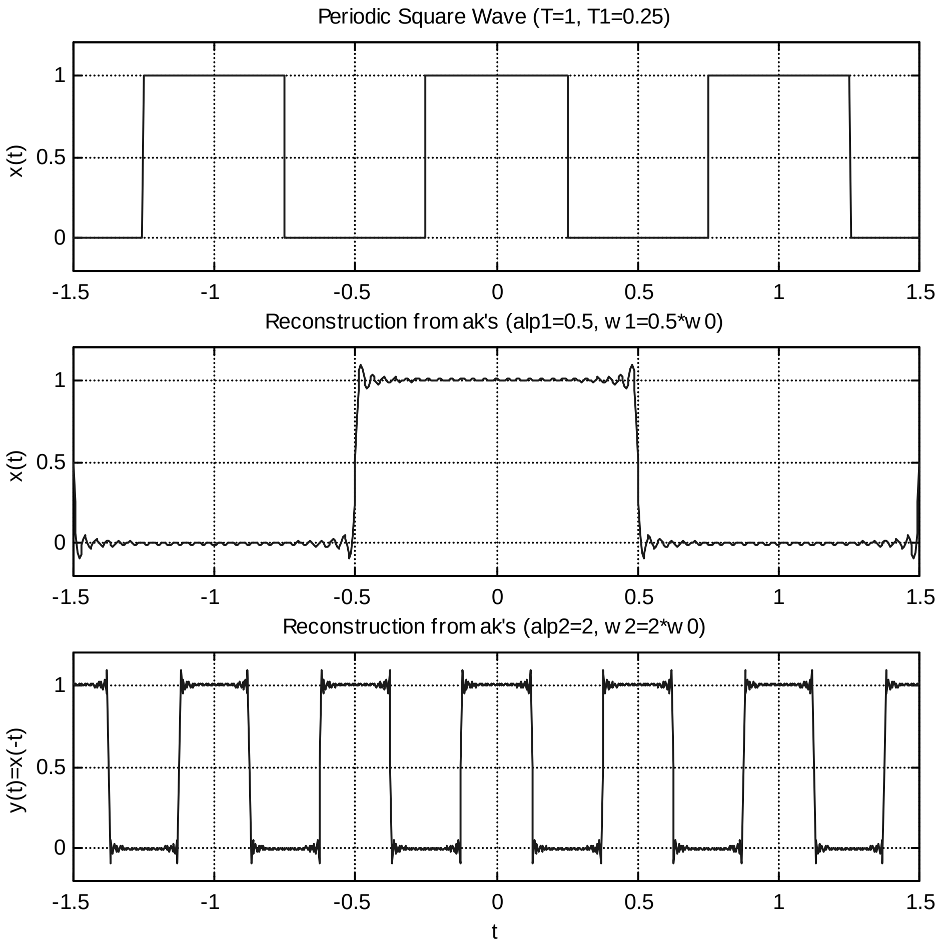

Figure 13.1 Periodic square wave signal with $T = 1$ and $T_1 = 0.25$ (upper panel), reconstructed signal using FS coefficients of the original signal at $w_1 = 0.5\,w_0 = \pi$ (middle panel), and a reconstruction using FS coefficients of the signal at $w_2 = 2\,w_0 = 4\,\pi$ (lower panel).

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% Project #13
% Title: Properties of Continuous Time Fourier Series
%        Time Scaling

% Generation of periodic square wave
t=-1.5:0.005:1.5;
xcos=cos(2*pi*t);
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xt=xcos>0;

% FS coefficients of periodic square wave
k=-50:50;
T=1;T1=0.25;
ak=sin(k*2*pi*(T1/T))./(k*pi);
% Manual correction for a0 -> ak(51)
ak(51)=2*T1/T;

% Time scaling parameters
alp1=0.5;
alp2=2;

% w's for the time scaled signals
w0=2*pi/T;
w1=alp1*w0;
w2=alp2*w0;

% Reconstruction from ak's with 101 terms (M=50)
xat1=zeros(1,length(t));
xat2=zeros(1,length(t));
for k=-50:50
   xat1=xat1 + ak(k+51)*exp(j*k*w1*t);
   xat2=xat2 + ak(k+51)*exp(j*k*w2*t);
end

figure(1);set(gcf,'defaultaxesfontsize',8)
subplot(3,1,1);plot(t,xt);ylabel('x(t)')
title('Periodic Square Wave (T=1, T1=0.25)')
set(gca,'ylim',[-0.2 1.2]);grid

subplot(3,1,2);plot(t,real(xat1));ylabel('x(t)')
title('Reconstruction from ak''s (alp1=0.5, w1=0.5*w0)')
set(gca,'ylim',[-0.2 1.2]);grid
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subplot(3,1,3);plot(t,real(xat2))
xlabel('t');ylabel('y(t)=x(-t)')
title('Reconstruction from ak''s (alp2=2, w2=2*w0)')
set(gca,'ylim',[-0.2 1.2]);grid
```