Project #2

Calculation of Signal Power

In this project we will see how to calculate average of power of continuous and discrete time periodic signals. Let us remember again that we can only simulate continuous time case in a digital environment like Matlab.

Part 1: Continuous Time Case

Here, we will simulate or generate one period of a continuous time cosine and calculate its average power by using the formula

$$P = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt\,. \tag{2.1}$$

We first generate a fine time axis extending from –1 to 0.995 as follows.

```
» t=-1:0.005:0.995;                                  MCL 1
```

We selected the period T to be 2 and to cover one period exactly, we selected the upper limit of our time interval as 0.995. Can you tell why?

Now, we form our cosine signal and plot it.

```
» xt=cos(2*pi*t/2);                                  MCL 2
» plot(t,xt)                                         MCL 3
```

We then element-wise square `xt` and take the absolute value.

```
» abs_xt_2=abs(xt.^2);                               MCL 4
```

Note the use of dot (.) operator above. When it encounters a multiplication, division or power operation, Matlab normally tries to do the corresponding matrix operations but we can override this and make those operations element-wise by putting a dot in front of them.

For a real signal, after squaring taking the absolute value does not make sense or is not necessary but for a complex signal this step is necessary as dictated by the formula (2.1). We also note that the order of squaring and absolute value operations can be reversed or changed. Therefore, what we did by `MCL 4` can also be accomplished by the following line.

```
» abs_xt_2=abs(xt).^2;
```

Now, we will use Euler's approximation to carry out the integral. This formula simply tells that we can approximate a definite integral using a sum.

$$\int_a^b x(t)dt \cong \sum_{n=0}^{N-1} x(a+n\Delta t)\Delta t, \quad \Delta t = \frac{b-a}{N} \tag{2.2}$$

In this technique, we divide the region over which we will do the integral into N parts or intervals, each of duration Δt, and assume that our function stays constant over those short intervals. Approximating a function in this (staircase) manner is depicted in Figure 2.1. As we increase the number of intervals N, the approximation gets better.

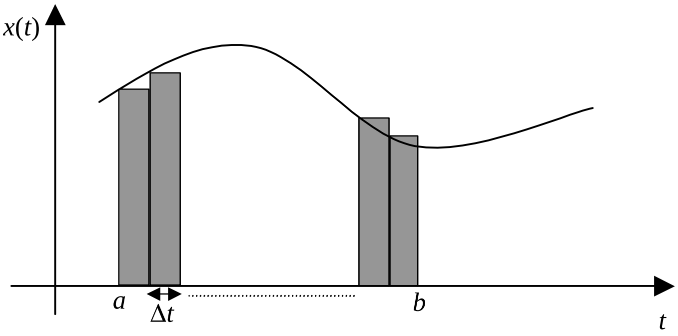

Figure 2.1 Euler's approximation.

Approximating integrals using sums as we did above, is a deep subject of numerical analysis or methods by itself, therefore we will not get into more detail here. It suffices for us to know that Euler's formula or technique is easy to implement and produces good results for almost all the signals that we will deal with, as long as N is selected large enough.

Back to our main subject, to approximate or compute the average power of our cosine signal, we first note that we have already selected Δt as 0.005. (Can you tell where we did this? Hint: Check `MCL 1`.) Let us now define this parameter and the period T.

```
» delta_t=0.005; T=2;                                   MCL 5
```

Finally we do the sum and multiply it with $\Delta t/T$.

```
» pxt=sum(abs_xt_2)*delta_t/T                           MCL 6

pxt =

    0.5000
```

Does the result look familiar?

Part 2: Discrete Time Case

We will now generate one period of a discrete time cosine and calculate its average power using the following formula

$$P = \frac{1}{N}\sum_{n=0}^{N-1} \left| x[n] \right|^2 . \tag{2.3}$$

We create a vector of indices first and form our discrete time cosine signal as follows.

```
» n=0:19;                                               MCL 7
» xn=cos(2*pi*n/20);                                    MCL 8
» stem(n,xn)                                            MCL 9
```

To cover exactly one period of the cosine we let the index run from 0 to 19, as the period N is 20.

The formula (2.3) to be implemented is already a sum and the calculation of average power is really easy this time, as we are not dealing with an integral.

```
» N=20; abs_xn_2=abs(xn.^2);                             MCL 10
» pxn=sum(abs_xn_2)/N                                    MCL 11

pxn =

    0.5000
```