Project #9

Determination of Fourier Series Representation of Continuous Time Periodic Signals and Converge of the Series

Once the Fourier series (FS) coefficients of a continuous time periodic signal is found analytically, using equation (8.4), we can reconstruct the signal using equation (8.1). Here we will demonstrate this concept for the periodic square wave signal, defined as

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < T/2 \end{cases}. \tag{9.1}$$

One sample of this signal with $T = 1$ and $T_1 = 0.25$ is shown in Figure 9.1. You can find the Matlab code segment that we used to generate Figure 9.1, along with other code segments of this project, at the end, packed in an mfile titled "project9.m". Note how we generated the square wave by thresholding a 1 Hz cosine, and also the indexing techniques that we used in this file.

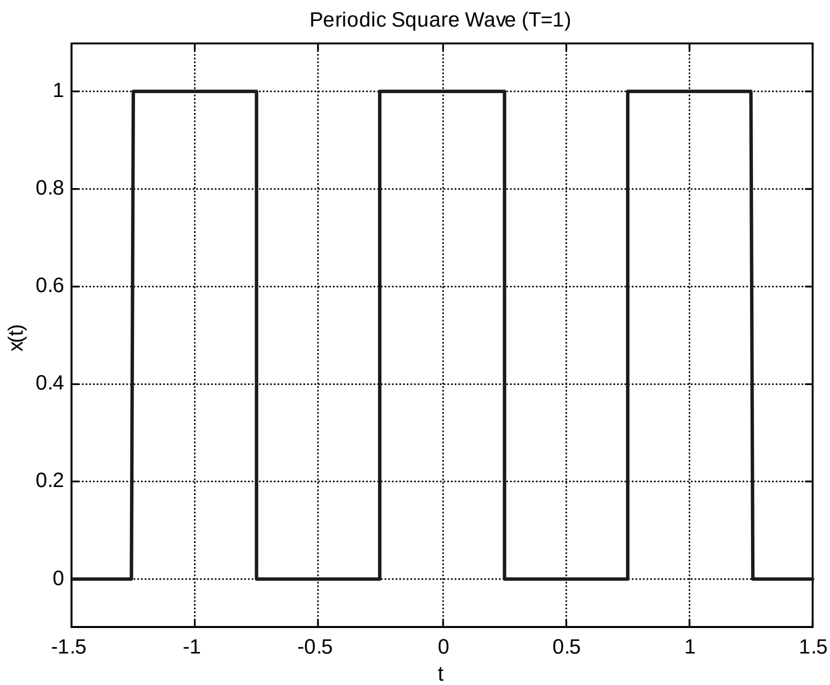

Figure 9.1 Periodic square wave with $T = 1$ and $T_1 = 0.25$.

The FS coefficients of periodic square wave can be found, using equation (8.4), as

$$a_0 = \frac{2T_1}{T}, \quad a_k = \frac{\sin(kw_0T_1)}{k\pi} = \frac{\sin(k2\pi(T_1/T))}{k\pi} \text{ for } k \neq 0. \qquad (9.2)$$

Let us first look at these coefficients to gain some insight (Figure 9.2). We plotted FS coefficients a_k's for three periodic square waves with $T = 1$, and T_1 = 1/4, 1/8 and 1/16. We observe that as T_1 gets lower the envelope of the coefficients, which is a sinc function, gets expanded.

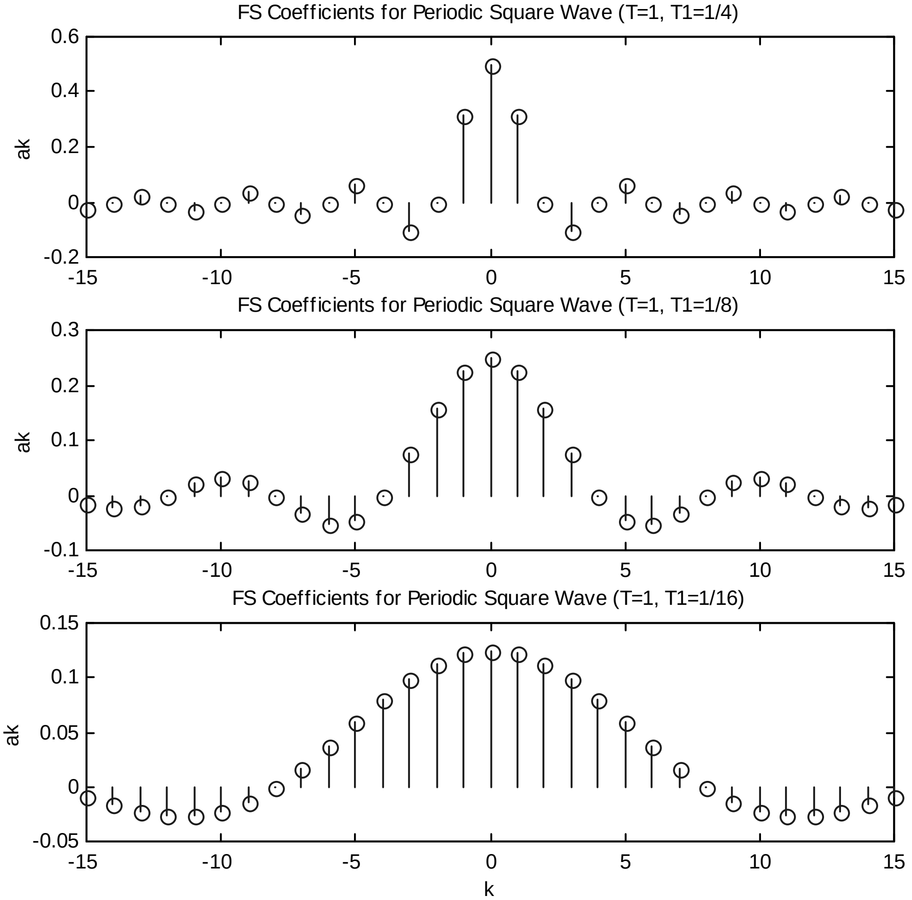

Figure 9.2 Fourier series coefficients a_k's for $k = -15 .. 15$, for three periodic square waves with period $T = 1$, and $T_1 = 1/4$ (upper panel), $T_1 = 1/8$ (middle panel) and $T_1 = 1/16$ (lower panel).

Now, using these coefficients in reconstruction equation (8.1), $x(t)=\sum_{k=-\infty}^{+\infty} a_k e^{jkw_0 t}$, repeated here for convenience, we can build our periodic square wave from its harmonics. We should note here that we cannot make the sum go from $k=-\infty$ to $k=+\infty$, instead, we will make it a partial sum by letting k go from $-M$ to M, where M will be 10, 20, and 100. We present the results of corresponding reconstructions with 21, 41, and 201 coefficients or terms in Figures 9.3 through 9.5.

Note the Gibbs phenomenon here, even we increase the number terms in the sum, oscillations at the edges are still there. The reconstruction cannot exactly follow the original signal at the discontinuities. Check the peak amplitude of oscillations, can you verify that it is really 9 % of the height of the discontinuity?) For further details about the Gibbs phenomenon, refer to your textbook.

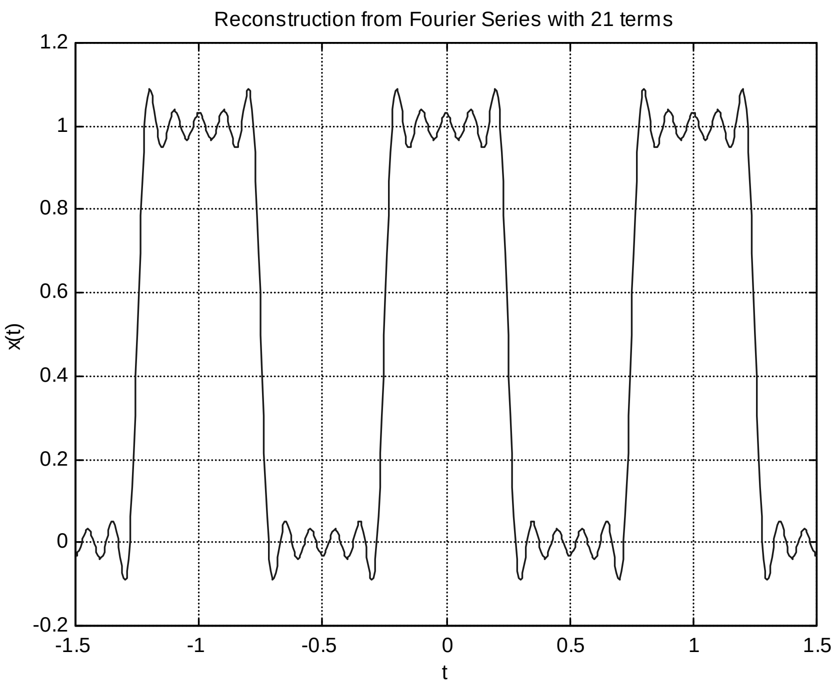

Figure 9.3 Reconstruction of periodic square wave ($T = 1$, $T_1 = T/4$) from its Fourier series coefficients using 21 terms (a_k's, $k = -10$ .. 10).

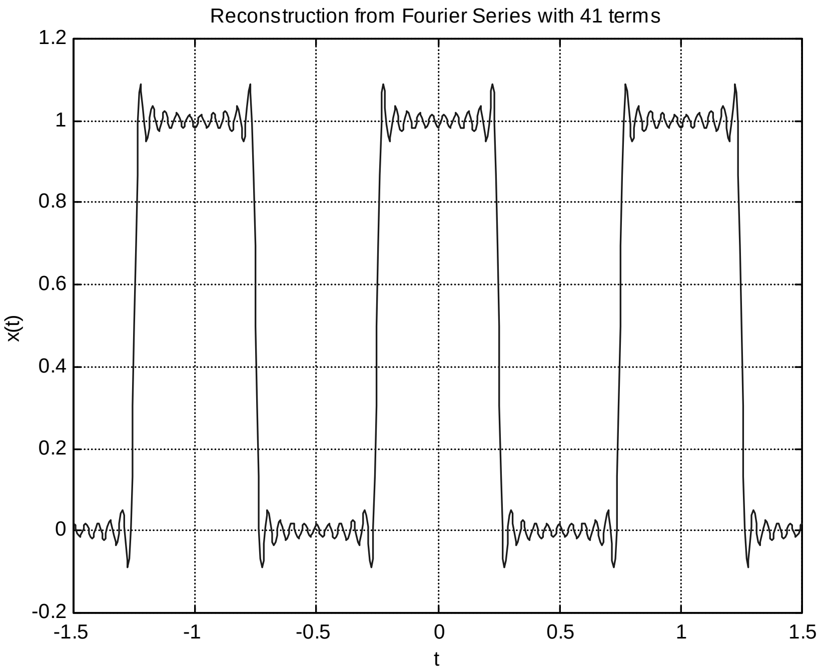

Figure 9.4 Reconstruction of periodic square wave ($T = 1$, $T_1 = T/4$) from its Fourier series coefficients using 41 terms (a_k's, $k = -20$ .. 20).

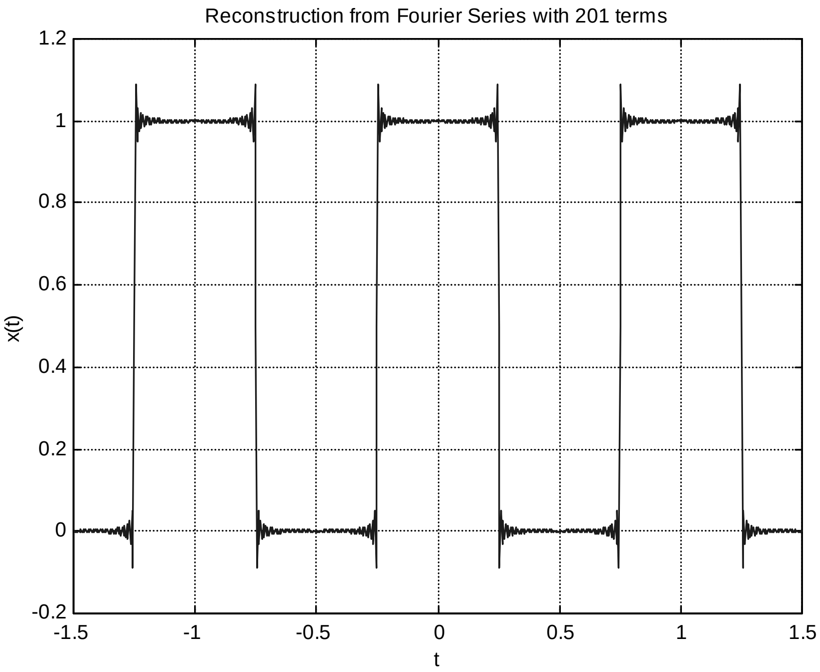

Figure 9.5 Reconstruction of periodic square wave ($T = 1$, $T_1 = T/4$) from its Fourier series coefficients using 201 terms (a_k's, $k = -100$ .. 100).

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% Project #9
% Title: Fourier Series Representation of Continuous Time
%        Periodic Signals

% Figure 9.1 (Generation of a periodic square wave with T=1
% by thresholding a 1Hz cosine)
figure(1)
set(gcf,'defaultaxesfontsize',9)
% This optional statement sets the fontsize property
% of axes objects to be created in the current figure
t=-1.5:0.005:1.5;
xcos=cos(2*pi*t);
% Thresholding, using Matlab’s relational operators, is done here
xpsqw=xcos>0; % Type “help relop” at Matlab prompt for details
plot(t,xpsqw);xlabel('t');ylabel('x(t)')
title('Periodic Square Wave (T=1)')
set(gca,'ylim',[-0.1 1.1]);grid

% Figure 9.2 (FS coefficients of periodic square wave)
figure(2);set(gcf,'defaultaxesfontsize',8)
k=-15:15;T=1;
T1=1/4;ak1=sin(k*2*pi*(T1/T))./(k*pi);
% Ignore the "divide by zero" warning that happens
% because k in the denominator hits 0. We will now do
% a manual correction for a0 -> ak1(16)
ak1(16)=2*T1/T;
subplot(3,1,1);stem(k,ak1);ylabel('ak')
title('FS Coefficients for Periodic Square Wave (T=1, T1=1/4)')

T1=1/8;ak2=sin(k*2*pi*(T1/T))./(k*pi);
ak2(16)=2*T1/T; % Manual correction for a0 -> ak2(16)
subplot(3,1,2);stem(k,ak2);ylabel('ak')
title('FS Coefficients for Periodic Square Wave... (T=1, T1=1/8)')

T1=1/16;ak3=sin(k*2*pi*(T1/T))./(k*pi);
ak3(16)=2*T1/T; % Manual correction for a0 -> ak3(16)
subplot(3,1,3);stem(k,ak3);xlabel('k');ylabel('ak')
title('FS Coefficients for Periodic Square Wave (T=1, T1=1/16)')
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% Figure 9.3 (Reconstruction with M=10)
figure(3);set(gcf,'defaultaxesfontsize',9)
T1=1/4;w0=2*pi/T;
M=10;k=-M:M;
ak=sin(k*2*pi*(T1/T))./(k*pi);
ak(M+1)=2*T1/T; % Manual correction for a0 -> ak(M+1)
x=zeros(1,length(t));
for k=-M:M
   x=x+ak(k+M+1)*exp(j*k*w0*t);
end
plot(t,real(x));grid;xlabel('t');ylabel('x(t)')
title('Reconstruction from Fourier Series with 21 terms')

% Figure 9.4 (Reconstruction with M=20)
figure(4);set(gcf,'defaultaxesfontsize',9)
M=20;k=-M:M;
ak=sin(k*2*pi*(T1/T))./(k*pi);
% Manual correction for a0 -> ak(M+1)
ak(M+1)=2*T1/T;
x=zeros(1,length(t));
for k=-M:M
   x=x+ak(k+M+1)*exp(j*k*w0*t);
end
plot(t,real(x));grid;xlabel('t');ylabel('x(t)')
title('Reconstruction from Fourier Series with 41 terms')

% Figure 9.5 (Reconstruction with M=100)
figure(5);set(gcf,'defaultaxesfontsize',9)
M=100;k=-M:M;
ak=sin(k*2*pi*(T1/T))./(k*pi);
% Manual correction for a0 -> ak(M+1)
ak(M+1)=2*T1/T;
x=zeros(1,length(t));
for k=-M:M
   x=x+ak(k+M+1)*exp(j*k*w0*t);
end
plot(t,real(x));grid;xlabel('t');ylabel('x(t)')
title('Reconstruction from Fourier Series with 201 terms')
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